TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

Advanced Placement Calculus AB

Mathematics Department

2018

(Last revision date: 1992)

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AP Calculus AB
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The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in any of its programs.

CORE VALUES AND BELIEFS

The Trumbull High School community engages in an environment conducive to learning which believes that all students will **read and write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION & PHILOSOPHY

AP Calculus AB is designed to develop mathematical knowledge conceptually, guiding students to connect topics and representations and to apply strategies and techniques to accurately solve diverse types of problems. The course will help students build enduring mathematical understandings by teaching students the how and why of mathematics. Students will come into this class having completed PreCalculus, and will become proficient with derivatives and their applications and integrals and their applications. The use of a graphing calculator is an integral part of the course. Students will be prepared to succeed on the Advanced Placement Calculus AB exam given in May.

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.
The successful mathematician will:

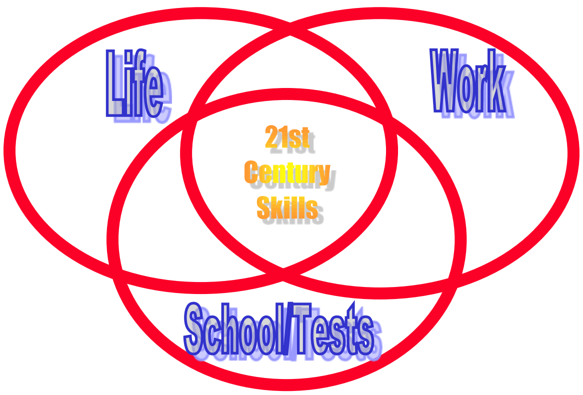
- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourge students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media and Technology Skills

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE GOALS

The following Course Goals derive from the 2010 Connecticut Core Standards for Mathematical Practice, which describe varieties of expertise that all teachers of mathematics will develop in their students. These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

At the completion of this course, students will:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In addition, six Mathematical Practices for AP Calculus have been articulated by the College Board.

MPAC 1: Reasoning with definitions and theorems

Students will:

a. use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
b. confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
c. apply definitions and theorems in the process of solving a problem;
d. interpret quantifiers in definitions and theorems (e.g., "for all," "there exists");
e. develop conjectures based on exploration with technology; and
f. produce examples and counterexamples to clarify understanding of definitions, to investigate whether converse of theorems are true or false, or to test conjectures.

MPAC 2: Connecting concepts

Students will:

a. relate the concept of a limit to all aspects of calculus;

b. use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
c. connect concepts to their visual representations with and without technology, and
d. identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes
Students will:
a. select appropriate mathematical strategies;
b. sequence algebraic/computational procedures logically;
c. complete algebraic/computational procedures correctly;
d. apply technology strategically to solve problems;
e. attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
f. connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations
Students will:
a. associate tables, graphs, and symbolic representations of functions;
b. develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
c. identify how mathematical characteristics of functions are related in different representations;
d. extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
e. construct one representational form from another (e.g., a table from a graph or a graph from given information); and
f. consider multiple representations (graphing, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building Notational Fluency
Students will:
a. know and use a variety of notation (e.g., $f'(x), y', \frac{dy}{dx}$);
b. connect notation to definition (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum)
c. connect notation to different representations (graphical, numerical, analytical, and verbal); and
d. assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating
Students will:
a. clearly present methods, reasoning, justification, and conclusions;
b. use accurate and precise language and notation;
c. explain the meaning of expression, notation, and results in terms of a context (including units);
d. explain the connection among concepts;
e. critically interpret and accurately report information provided by technology; and
f. analyze, evaluate, and compare the reasoning of others.

The following Course Goals derive from the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions. 5b. Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making. 5c. Students break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

COURSE ENDURING UNDERSTANDINGS

Students will understand that . . .

- the concept of a limit can be used to understand the behavior of a function.
- continuity is a key property of functions that is defined using limits.
- the derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
- a function's derivative, which is itself a function, can be used to understand the behavior of the function.
- the derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
- the Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.
- antidifferentiation is the inverse process of differentiation.
- the definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
- the Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
- the definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
- antidifferentiation is an underlying concept involved in solving seperable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.

COURSE ESSENTIAL QUESTIONS

- What is a limit, and how can it be interpreted?
- What is a derivative, and how can it be applied?
- What is an integral, and how can it be applied?

COURSE KNOWLEDGE & SKILLS

Students will know . . .

- that, given a function f, the limit of $f(x)$ as $x$approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x\to c} f(x) = R$.
- that he concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- that the limit might not exist for some functions at particular values of x.Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- that numerical and graphical information can be used to estimate limits.
- that limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- that the limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
- that limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hôpital's Rule.
- that asymptotic and unbounded behavior of functions can be explained and described using limits.
- that relative magnitudes of functions and their rates of change can be compared using limits.
- that a function f is continuous at $x = c$ provided that $f(c)$exists, $\lim_{x\to c} f(x)$ exists, and $\lim_{x\to c} f(x) = f(c)$.
- that polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
- that types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- that continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
- that the difference quotients $\frac{f(a+h)-f(a)}{h}$and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- that the instantaneous rate of change of a function at a point can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
- that the derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists.
- that, for $y = f(x)$, notations for the derivative include $\frac{dy}{dx}, f'(x)$,and y'.
- that the derivative can be represented graphically, numerically, analytically, and verbally.
- that the derivative at a point can be estimated from information given in tables or graphs.
- that direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.

- that specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- that sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- that the chain rule provides a way to differentiate composite functions.
- that the chain rule is the basis for implicit differentiation.
- that the chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
- that differentiating f' produces the second derivative f'', provided the derivative of f'exists; repeating this process produces higher order derivatives of f.
- that higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$,and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.
- that first and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- that key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- that key features of the graphs of f, f', and f'' are related to one another.
- that a continuous function may fail to be differentiable at a point in its domain.
- that if a function is differentiable at a point, then it is continuous at that point.
- that the unit for $f'(x)$ is the unit for f divided by the unit for x.
- that the derivative of a function can be interpreted as the instantaneous rate of change with respect to the independent variable.
- that the derivative at a point is the slope of the line tangent to a graph at that point on the graph.
- that the tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- that the derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- that the derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- that the derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- that the derivative can be used to express information about rates of change in applied contexts.
- that, if a function $f$is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.
- that an antiderivative of a function f is a function g whose derivative is f.
- that differentiation rules provide the foundation for finding antiderivatives.

- that a Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- that the definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)\,dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)\,dx = \lim_{\max\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where x_i^* is a value in the i^{th} subinterval, Δx_i is the width of the i^{th} subinterval, n is the number of subintervals, and $\max\Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the i^{th} subinterval.
- that the information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- that definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
- that definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann Sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- that, in some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- that properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- that the definition of the definite integral may be extended to functions with removable or jump discontinuities.
- that the definite integral can be used to define new functions; for example, $f(x) = \int_0^x e^{-t^2}\,dt$.
- that, if f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, where x is between a and b.
- that graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$.
- that the function defined by $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f.
- that, if f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.
- that the notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$,and $\int f(x)dx$ is called an indefinite integral of the function f.
- that many functions do not have closed form antiderivatives.
- that techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, and substitution of variables.
- that a function defined as an integral represents an accumulation of a rate of change.
- that the definite integral of a rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- that the limit of an approximating Riemann sum can be interpreted as a definite integral.
- that areas of certain regions in the plane can be calculated with definite integrals.
- that solutions to differential equations are functions or families of functions.

- that derivatives can be used to verify that a function is a solution to a given differential equation.
- that slope fields provide visual clues to the behavior of solutions to first order differential equations.
- that the average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.
- that, for a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- that volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
- that the definite integral can be used to express information about accumulation and net change in many applied contexts.
- that antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, and exponential growth and decay.
- that some differential equations can be solved by separation of variables.
- that solutions to differential equations may be subject to domain restrictions.
- that the function F defined by $F(x) = c + \int_a^x f(t)\,dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)\,dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.
- that the model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.

Students will be able to . . .

- express limits symbolically using correct notation.
- interpret limits expressed symbolically.
- estimate limits of functions.
- determine limits of functions.
- deduce and interpret behavior of functions using limits.
- analyze functions for intervals of continuity or points of discontinuity.
- determine the applicability of important calculus theorems using continuity.
- identify the derivative of a function as the limit of a difference quotient.
- estimate derivatives.
- calculate derivatives.
- determine higher order derivatives.
- use derivatives to analyze properties of a function.
- recognize the connection between differentiability and continuity.
- interpret the meaning of a derivative within a problem.
- solve problems involving the slope of a tangent line.
- solve problems involving related rates, optimization, and rectilinear motion.
- solve problems involving rates of change in applied contexts.
- apply the Mean Value Theorem to describe the behavior of a function over an interval.
- recognize antiderivatives of basic functions.

- interpret the definite integral as the limit of a Riemann sum.
- express the limit of a Riemann sum in integral notation.
- approximate a definite integral.
- calculate a definite integral using areas and properties of definite integrals.
- analyze functions defined by an integral.
- calculate antiderivatives.
- evaluate definite integrals.
- interpret the meaning of a definite integral within a problem.
- apply definite integrals to problems involving area and volume.
- verify solutions to differential equations.
- estimate solutions to differential equations.
- apply definite integrals to problems involving the average value of a function.
- apply definite integrals to problems involving motion.
- use the definite integral to solve problems in various contexts.
- analyze differential equations to obtain general and specific solutions.
- interpret, create, and solve differential equations from problems in context.

COURSE SYLLABUS

Course Name

Advanced Placement Calculus AB

Level

Advanced Placement

Prerequisites

Completion of Honors PreCalculus, or Grade of B+ or better and teacher recommendation in ACP PreCalculus.

Materials Required

TI-84 Plus graphing calculator

General Description of the Course

The Advanced Placement Calculus AB curriculum follows the curriculum prescribed by the College Board. The first semester emphasizes a thorough study of derivatives. Students become proficient at both explicit and implicit derivatives of polynomial, rational, trigonometric, logarithmic, and exponential functions. Derivative applications are studied through motion, curve fitting, extreme, and related rates. The second semester stresses the study of the integral. Applications are stressed through the investigation of volumes, length of curves, volumes of solids of revolution, and surface area. The use of the graphing calculator is an integral part of the course. Summer work is required.

Assured Assessments

Formative Assessments:

Formative assessments can include, but are not limited to:

- Problem sets (Units 1, 2, 3, 4, 5)
- Quizzes (Units 1, 2, 3, 4, 5)
- Homework (Units 1, 2, 3, 4, 5)
- Problems of the day (Units 1, 2, 3, 4, 5)

Summative Assessments:

- Common end-of-unit tests (Units 1, 2, 3, 4, 5)
- Common midterm examination
- Common final examination

Core Text

- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10^{th} ed. Boston: Cengage, 2015. Print.

UNIT 1
Limits

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2016 College Board Curriculum Framework for Advanced Placement Calculus.

1.1A(a)	Express limits symbolically using correct notation.
1.1A(b)	Interpret limits expressed symbolically.
1.1B	Estimate limits of functions.
1.1C	Determine limits of functions.
1.1D	Deduce and interpret behavior of functions using limits.
1.2A	Analyze functions for intervals of continuity or points of discontinuity.
1.2B	Determine the applicability of important calculus theorems using continuity.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How can a limit be used to understand the behavior of functions?
- Why is continuity a key property of functions, and how can limits be used to describe it?
- How can a limit be evaluated algebraically?
- How can a limit be evaluated graphically?
- How can a limit be evaluated analytically?

Scope and Sequence

1. Expressing limits symbolically using correct notation
2. Interpreting limits expressed symbolically
3. Estimating limits of functions
4. Determining limits of functions
5. One-sided limits
6. Limits to infinity and limit at infinity
7. Deducing and interpreting behavior of functions using limits
8. Analyzing functions for interval of continuity or points of discontinuity

9. Determining the applicability of important calculus theorems using continuity

Assured Assessments

Formative Assessment:
Students will participate in various problem sets, quizzes, homework, and problems of the day throughout the unit.

Summative Assessment:
Students will take an end-of-unit test scored via a common scoring guide.

Resources

Core

- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10^{th} ed. Boston: Cengage, 2015. Print.

Supplemental

- Worksheets created with Kuta Software, a free online resource
- Released AP Calculus exams from the College Board

Time Allotment

- Approximately 30 school days

UNIT 2
Derivatives

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2016 College Board Curriculum Framework for Advanced Placement Calculus.

2.1A	Identify the derivative of a function as the limit of a difference quotient.
2.1B	Estimate derivatives.
2.1C	Calculate derivatives.
2.1D	Determine higher order derivatives.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How does the concept of a limit lead to a derivative?
- How does one differentiate a polynomial function using the power rule?
- How does one differentiate a rational function using the quotient rule?
- How does one differentiate a product of functions, including trigonometric functions, using the product rule?
- How does one differentiate a composition of functions using the chain rule?
- How does one differentiate implicit equations?

Scope and Sequence

1. Identifying the derivative of a function as the limit of a difference quotient
2. The tangent line problem
3. Average and instantaneous rates of change
4. Definitions of the derivative
5. Equations of tangent lines
6. Estimating derivatives
7. Calculating derivatives
8. Determining higher order derivatives

Assured Assessments

Formative Assessment:
Students will participate in various problem sets, quizzes, homework, and problems of the day throughout the unit.

Summative Assessment:
Students will take an end-of-unit test scored via a common scoring guide.

Resources

Core
- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10th ed. Boston: Cengage, 2015. Print.

Supplemental
- Worksheets created with Kuta Software, a free online resource
- Released AP Calculus exams from the College Board

Time Allotment

- Approximately 30 school days

UNIT 3
Applications of Derivatives

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2016 College Board Curriculum Framework for Advanced Placement Calculus.

2.2A	Use derivatives to analyze properties of a function.
2.2B	Recognize the connection between differentiability and continuity.
2.3A	Interpret the meaning of a derivative within a problem.
2.3B	Solve problems involving the slope of a tangent line.
2.3C	Solve problems involving related rates, optimization, and rectilinear motion.
2.3D	Solve problems involving rates of change in applied contexts.
2.4A	Apply the Mean Value Theorem to describe the behavior of a function over an interval.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What information can be determined from the derivative to help sketch the graph of a function?
- How can the derivative be used to solve problems involving instantaneous rates of change?
- How can the derivative be used to solve problems requiring optimization?
- How can the derivative be used to solve problems involving position, velocity, and acceleration?

Scope and Sequence

1. Interpreting the meaning of a derivative within a problem
2. Related rates, optimization, and rectilinear motion
3. Solving problems involving rates of change in applied contexts
4. Extrema
5. Mean Value Theorem and Rolle's Theorem

6. Increasing and decreasing functions
7. First Derivative Test
8. Concavity
9. Second Derivative Test

Assured Assessments

Formative Assessment:
Students will participate in various problem sets, quizzes, homework, and problems of the day throughout the unit.

Summative Assessment:
Students will take an end-of-unit test scored via a common scoring guide.

Resources

<u>Core</u>

- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10th ed. Boston: Cengage, 2015. Print.

<u>Supplemental</u>

- Worksheets created with Kuta Software, a free online resource
- Released AP Calculus exams from the College Board

Time Allotment

- Approximately 45 school days

UNIT 4
Integrals and the Fundamental Theorem of Calculus

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2016 College Board Curriculum Framework for Advanced Placement Calculus.

3.1A	Recognize antiderivatives of basic functions.
3.2B(a)	Interpret the definite integral as the limit of a Riemann sum.
3.2A(b)	Express the limit of a Riemann sum in integral notation.
3.2B	Approximate a definite integral.
3.2C	Calculate a definite integral using areas and properties of definite integrals.
3.3A	Analyze functions defined by an integral.
3.3B(a)	Calculate antiderivatives.
3.3B(b)	Evaluate definite integrals.
3.4A	Interpret the meaning of a definite integral within a problem.
3.4D	Apply definite integrals to problems involving area and volume.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What is an integral?
- How does the process of antidifferentiation relate to the process of differentiation?
- What is a Riemann sum, and how can it be used to calculate the definite integral?
- How does the concept of a limit lead to the area under the curve?
- What is the Fundamental Theorem of Calculus, and how does it connect integration and differentiation?

Scope and Sequence

1. Antiderivatives of basic functions
2. Riemann sums (left, right, midpoint)

3. Trapezoidal sums
4. Expressing the limit of a Riemann sum in integral notation
5. Definite integral
6. Area under the curve
7. Analyzing functions defined by an integral
8. The Fundamental Theorem of Calculus
9. The Second Fundamental Theorem of Calculus

Assured Assessments

Formative Assessment:
Students will participate in various problem sets, quizzes, homework, and problems of the day throughout the unit.

Summative Assessment:
Students will take an end-of-unit test scored via a common scoring guide.

Resources

Core

- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10th ed. Boston: Cengage, 2015. Print.

Supplemental

- Worksheets created with Kuta Software, a free online resource
- Released AP Calculus exams from the College Board

Time Allotment

- Approximately 30 school days

UNIT 5
Applications of Integration

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2016 College Board Curriculum Framework for Advanced Placement Calculus.

2.3E	Verify solutions to differential equations.
2.3F	Estimate solutions to differential equations.
3.4B	Apply definite integrals to problems involving the average value of a function.
3.4C	Apply definite integrals to problems involving motion.
3.4D	Apply definite integrals to problems involving area and volume.
3.4E	Use the definite integral to solve problems in various contexts.
3.5A	Analyze differential equations to obtain general and specific solutions.
3.5B	Interpret, create, and solve differential equations from problems in context.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker (Standard 5)	Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How does one use integration to find volume?
- What is a differential equation?
- How does one use initial conditions to find particular solutions of differential equations?
- How does one use slope fields to obtain the solutions to a first order differential equation?
- How can antidifferentiation be used to solve separable differential equations?

Scope and Sequence

1. Interpreting the meaning of a definite integral within a problem
2. Average value of a function
3. Applying definite integrals to problems involving motion
4. The definite integral and volume
5. Using the definite integral to solve problems in various contexts

6. Differential equations
7. General and specific solutions
8. Interpreting, creating, and solving differential equations from problems in context
9. Slope fields

Assured Assessments

Formative Assessment:
Students will participate in various problem sets, quizzes, homework, and problems of the day throughout the unit.

Summative Assessment:
Students will take an end-of-unit test scored via a common scoring guide.

Resources

Core

- Larson, Ron, and Bruce Edwards. *Calculus of a Single Variable*. 10th ed. Boston: Cengage, 2015. Print.

Supplemental

- Worksheets created with Kuta Software, a free online resource
- Released AP Calculus exams from the College Board

Time Allotment

- Approximately 30 school days

CULMINATING ACTIVITY

Following the Advanced Placement Examination, students will complete a research project with paper and presentation components. The project is designed for students to explore mathematical concepts that they have not typically had the chance to during their high school mathematics career. Students can choose to research various applications of math, new concepts, or a specific mathematician. They can also choose to work independently, in pairs, or, with permission, in a group of 3.

The paper should reflect the research done and should be approximately 4-5 pages in length with a proper Works Cited based on 5 reliable sources. The presentation should be approximately 10-15 minutes in length, and can take several forms, but cannot include videos of others speaking for more than 5 minutes.

Time Allotment

- Approximately 15 school days

TEACHER GUIDE

Unit One: Limits

Sections:
1.1 A Preview of Calculus
1.2 Find Limits Graphically and Numerically
1.3 Evaluating Limit Analytically
1.4 Continuity and One-Sided Limits
1.5 Infinite Limits
3.5 Limits at Infinity

Unit Two: Derivatives

2.1 The Derivative and the Tangent Line Problem
2.2 Basic Differentiation Rules and Rates of Changes
2.3 Product and Quotient Rules and Higher Order Derivatives
2.4 The Chain Rule
5.1 The Natural Logarithmic Function: Differentiation
5.3 Inverse Functions
5.4 Exponential Functions: Differentiation (differentiation only)
5.6 Inverse Trigonometric Differentiation
2.5 Implicit Differentiation

Unit Three: Applications of Derivatives

2.6 Related Rates
3.1 Extrema on an Interval
3.2 Rolle's Theorem and Mean Value Theorem
3.3 Increasing and Decreasing Functions and the First Derivative Test
3.4 Concavity and the Second Derivative Test
3.6 A Summary of Curve Sketching
3.7 Optimization Problems

Unit Four: Integrals and the Fundamental Theorem of Calculus

4.1 Antiderivatives and Indefinite Integration

***Midterm exam**

Unit Four: Integrals and the Fundamental Theorem of Calculus

4.2 Area
4.3 Riemann Sums and Definite Integrals
4.4 The Fundamental Theorem of Calculus
4.5 Integration by Substitution
5.2 The National Logarithmic Function: Integration
5.4 Exponential Functions: Integration (integration only)
5.7 Inverse Trigonometric Functions: Integration
4.6 Numerical Integration (trapezoidal sum only)

Unit Five: Applications of Integration

6.1 Slope Fields (omitting Euler's Method)

6.2 Differential Equations: Growth and Decay
6.3 Separation of Variables (omitting the logistic equation)
7.1 Area of a Region between Two Curves
7.2 Volume: The Disk Method
8.7 Indeterminate Forms and L'Hôpital's Rule

***Final exam**

COURSE CREDIT

One credit in Mathematics
One class period daily for a full year

PREREQUISITES

Completion of Honors PreCalculus, or Grade of B+ or better and teacher recommendation in ACP PreCalculus.

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department- and teacher-prepared materials

TI-84 Plus graphing calculators

College Board website including past Advanced Placement Calculus AB tests

CURRENT REFERENCES

College Board: Advanced Placement Calculus AB:

https://apcentral.collegeboard.org/courses/ap-calculus-ab?course=ap-calculus-ab

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric (attached)
- Trumbull High School School-Wide Problem-Solving Rubric (attached)
- Trumbull High School School-Wide Independent Learning and Thinking Rubric (attached)

Trumbull High School School-Wide Writing Rubric

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_______	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_______	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_______	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_______	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Trumbull High School School-Wide Problem-Solving Rubric

Category/ Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_______	• Student demonstrates clear understanding of the problem and the complexities of the task	• Student demonstrates sufficient understanding of the problem and most of the complexities of the task	• Student demonstrates some understanding of the problem but requires assistance to complete the task	• Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_______	• Student gathers compelling information from multiple sources including digital, print, and interpersonal	• Student gathers sufficient information from multiple sources including digital, print, and interpersonal	• Student gathers some information from few sources including digital, print, and interpersonal	• Student gathers limited or no information
Reasoning and Strategies X_______	• Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	• Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	• Student demonstrates some critical thinking skills to develop a plan integrating some strategies	• Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_______	• Solution shows deep understanding of the problem and its components • Solution shows extensive use of 21st-century technology skills	• Solution shows sufficient understanding of the problem and its components • Solution shows sufficient use of 21st-century technology skills	• Solution shows some understanding of the problem and its components • Solution shows some use of 21st-century technology skills	• Solution shows limited or no understanding of the problem and its components • Solution shows limited or no use of 21st-century technology skills

Trumbull High School School-Wide
Independent Learning and Thinking Rubric

Category/ Weight	**Exemplary 4**	**Goal 3**	**Working Toward Goal 2**	**Needs Support 1-0**
Proposal X_______	• Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work	• Student demonstrates initiative by generating appropriate questions, creating original projects/work	• Student demonstrates some initiative by generating questions, creating appropriate projects/work	• Student demonstrates limited or no initiative by generating few questions and creating projects/work
Independent Research & Development X_______	• Student is analytical, insightful, and works independently to reach a solution	• Student is analytical, and works productively to reach a solution	• Student reaches a solution with direction	• Student is unable to reach a solution without consistent assistance
Presentation of Final Product X_______	• Presentation shows compelling evidence of an independent learner and thinker • Solution shows deep understanding of the problem and its components • Solution shows extensive and appropriate application of 21st-century skills	• Presentation shows clear evidence of an independent learner and thinker • Solution shows adequate understanding of the problem and its components • Solution shows adequate application of 21st-century skills	• Presentation shows some evidence of an independent learner and thinker • Solution shows some understanding of the problem and its components • Solution shows some application of 21st-century skills	• Presentation shows limited or no evidence of an independent learner and thinker • Solution shows limited or no understanding of the problem and its components • Solution shows limited or no application of 21st-century skills