

**TRUMBULL PUBLIC SCHOOLS**  
Trumbull, Connecticut

**Multivariable Calculus**  
Mathematics Department  
Trumbull High School

**2018**

**Curriculum Writing Team**

**Fran Basbagill**                      **Department Chair**

**Elizabeth Capobianco**           **Mathematics Teacher, Trumbull High School**

**Jonathan S. Budd, Ph.D., Assistant Superintendent of Curriculum, Instruction, & Assessments**

Multivariable Calculus            Property of Trumbull Public Schools

# Multivariable Calculus

## Table of Contents

Core Values & Beliefs .....	2
Introduction .....	2
Philosophy .....	2
Course Description .....	3
Course Goals .....	4
Course Enduring Understandings .....	5
Course Essential Questions .....	6
Course Knowledge & Skills .....	6
Course Syllabus .....	7
Unit 1: Vectors and the Geometry of Space .....	8
Unit 2: Vector-Valued Functions .....	10
Unit 3: Functions of Several Variables .....	12
Unit 4: Multiple Integration .....	14
Unit 5: Vector Analysis .....	16
Culminating Activity .....	18
Teacher Guide .....	19
Course Credit .....	21
Prerequisites.....	21
Text.....	21
Supplementary Materials/Resources/Technology .....	21
Current References .....	21
Assured Student Performance Rubrics .....	21

The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in any of its programs.

## CORE VALUES AND BELIEFS

The Trumbull School Community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

## INTRODUCTION

Multivariable Calculus is designed for the student who has successfully completed Advanced Placement / Early College Experience Calculus BC. Multivariable Calculus is an extension of calculus with one variable to calculus with functions of several variables. In Multivariable Calculus, students will become proficient with the differentiation and integration of functions involving multiple variables as well as applications in the real world such as volume and surface area.

## PHILOSOPHY

**Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.**

The Trumbull Mathematics Program embraces these goals for all students.

The successful mathematician will:

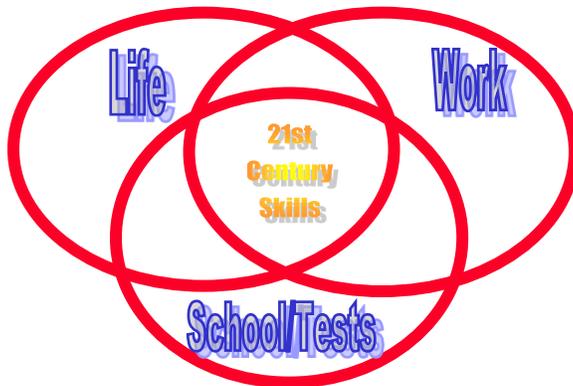
- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21<sup>st</sup> century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourage students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media And Technology Skills

1. Use real-world digital and other research tools to access, evaluate and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

## **COURSE DESCRIPTION**

Multivariable Calculus is a college-level course that follows Advanced Placement / Early College Experience Calculus BC. The course emphasizes a thorough study of vectors, surfaces in space, vector-valued functions, functions of several variables, multiple integrations, and vector analysis. Students will become proficient at vector operations including the dot product and cross product and their applications, rectangular coordinates, cylindrical coordinates, and spherical coordinates. Students will learn operations and applications of vector-valued functions including differentiation, integration, velocity, acceleration, tangent vectors, and normal vectors. Realizing that many real-life quantities are functions of two more variables, students will understand the following implementations of functions of several variables: limits, continuity, derivatives, and integration. The goal is to learn, understand, and be able to work with the main ideas of multivariable calculus. Students should not only be able to work through problems similar to

ones seen in the homework, but should also have the ability to go beyond, presenting their knowledge in a clear and coherent manner in problem sets related to each section. A graphing calculator is required for this course. Students will receive AP weight added to their GPA.

## **COURSE GOALS**

The Standards for Mathematical Practice describe varieties of expertise that all teachers of mathematics will develop in their students.

These practices rest on important “processes and proficiencies” that have long been valued in mathematics education.

### **1. Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

### **2. Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### **3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

### **4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and

formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### **5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

### **6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### **7. Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

### **8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## **COURSE ENDURING UNDERSTANDINGS**

Students will understand that . . .

- we live in a three-dimensional world.
- living in a three-dimensional world requires mathematical models that involve multivariables.
- various multivariable functions can be explored and understood verbally, numerically, algebraically, and visually.

## **COURSE ESSENTIAL QUESTIONS**

- How can the concepts of multivariable calculus help us understand the physical world around us?
- How can a previous knowledge of single-variable calculus be used to understand the concepts of multivariable calculus?

## **COURSE KNOWLEDGE & SKILLS**

Students will understand . . .

- the applications of the derivative and the integral in multivariable calculus.
- vector functions and coordinate planes.
- functions of two or more variables from verbal, numerical, visual, and algebraic points of view

Students will be able to . . .

- perform vector operations and interpret the results geometrically.
- use vector-valued functions to analyze projectile motion.
- find directional derivatives and gradients of functions.
- find the volume, center of mass, and moments of inertia of a solid region.
- sketch a vector field, determine whether a vector field is conservative, find a potential function, find curl, and find divergence.

# COURSE SYLLABUS

**Course Name**

Multivariable Calculus

**Level**

Advanced Placement

**Prerequisites**

Completion of Advanced Placement / Early College Experience Calculus BC with a B or better and teacher recommendation.

**Materials Required**

TI-84 graphing calculator

**General Description of the Course**

Multivariable Calculus is a college-level course that follows Advanced Placement / Early College Experience Calculus BC. The course emphasizes a thorough study of vectors, surfaces in space, vector-valued functions, functions of several variables, multiple integrations, and vector analysis. Students will become proficient at vector operations including the dot product and cross product and their applications, rectangular coordinates, cylindrical coordinates, and spherical coordinates. Students will learn operations and applications of vector-valued functions including differentiation, integration, velocity, acceleration, tangent vectors, and normal vectors. Realizing that many real-life quantities are functions of two more variables, students will understand the following implementations of functions of several variables: limits, continuity, derivatives, and integration. The goal is to learn, understand, and be able to work with the main ideas of multivariable calculus. Students should not only be able to work through problems similar to ones seen in the homework, but should also have the ability to go beyond, presenting their knowledge in a clear and coherent manner in problem sets related to each section. A graphing calculator is required for this course. Students will receive AP weight added to their GPA.

**Assured Assessments**

Students will be evaluated by their performance on tests, quizzes, homework, problem sets, other formative and summative assessments, and midterm and final examinations.

**Core Text**

Larson, Ron, and Bruce H. Edwards. *Calculus: AP Edition*. 9<sup>th</sup> ed. Boston: Brooks/Cole, 2010. Print.

# Unit 1: Vectors and the Geometry of Space

## Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Calculus Curriculum Guide.

- Write vectors, perform basic vector operations, and represent vectors graphically.
- Plot points in a three-dimensional coordinate system and analyze vectors in space.
- Find the dot product of two vectors.
- Find the cross product of two vectors.
- Find equations of lines and planes in space.
- Write and recognize equations of cylindrical and quadric surfaces.
- Use cylindrical and spherical coordinates to represent surfaces in space.

## Essential Questions

- What is a dot product and how can it be used to measure the work done by a force?
- What is a cross product and what are its applications in physics and engineering?
- How do the rectangular, cylindrical, and spherical coordinate systems relate to each other in three-space?

## Content (Scope and Sequence)

- Component form of a vector
- Vector operations
- Three-dimensional rectangular coordinate system
- Dot product and cross product
- Projection and vector components
- Area of a parallelogram using cross product
- Volume of a parallelepiped
- Distance between points, lines, and planes
- Equations of cylindrical and quadric surfaces
- Cylindrical coordinate system
- Spherical coordinate system

## Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

## Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.

2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

### **Assured Experiences**

Problem Sets

Homework

Mid-Unit Quizzes

Tests

### **Time Allocation**

Approximately 7 weeks

## Unit 2: Vector-Valued Functions

### Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Calculus Curriculum Guide.

- Know how to use vector-valued functions to analyze projectile motion.
- Apply concepts of limits and continuity to vector-valued functions.

### Essential Questions

- What is a vector function?
- What is a derivative and integral of a vector function?
- What is a space curve and how do we measure its length and curvature?
- How can the ideas of a tangent and normal vectors and curvature be used in physics to study the motion of an object, including its velocity and acceleration, along a space curve?

### Content (Scope and Sequence)

- Limits and continuity of vector-valued functions
- Differentiation and integration of vector-valued functions
- Velocity and acceleration of vector-valued functions
- Unit tangent vectors
- Tangential and normal components of acceleration
- Arc length of a space curve
- Curvature of a curve at a point on the curve

### Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

### Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

## **Assured Experiences**

Problem Sets

Homework

Mid-Unit Quizzes

Tests

## **Time Allocation**

Approximately 7 weeks

## Unit 3: Functions of Several Variables

### Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Calculus Curriculum Guide.

- Explore and understand verbal, numerical, algebraic, and visual representations of various multivariable functions.
- Investigate limits, continuity, and differentiation of functions of two independent variables, defining and applying the gradient.

### Essential Questions

- What does it mean for a function in three-space to be continuous?
- What is a partial derivative, and how is it interpreted?
- How is the chain rule applied when taking derivatives of functions of two variables?
- What is a directional derivative?
- What is a gradient vector, and what meaning does it have?
- How does one calculate the minima and maxima values of a function of two variables?
- What are the applications of the maximum or minimum value of a function?

### Content (Scope and Sequence)

- Sketching the graph of a function of two variables
- Sketching level curves for a function of two or three variables
- Extending the concept of continuity to a function of two or three variables
- Finding and using partial derivatives of a function of two or more variables
- Finding higher-order partial derivatives of a function of two or more variables
- Extending the concept of differentiability to a function of two variables
- Using the chain rule for functions of several variables
- Finding partial derivatives implicitly
- Finding and using directional derivatives of a function of two or more variables
- Finding and using the gradient of a function of two or more variables
- Finding equations of tangent planes and normal lines to surfaces
- Finding the angle of inclination of a plane in space
- Finding absolute and relative extrema of a function of two variables
- Using the second partials test to find relative extrema of a function of two variables
- Solving optimization involving functions of several variables
- Understanding the method of Lagrange multipliers
- Using Lagrange multipliers to solve constrained optimization problems

### Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling,

question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

## **Technology Competency Standards**

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

## **Assured Experiences**

Problem Sets

Homework

Mid-Unit Quizzes

Tests

## **Time Allocation**

Approximately 7 weeks

## Unit 4: Multiple Integration

### Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Calculus Curriculum Guide.

- Use double and triple integrals to compute volumes, surface areas, masses, and centroids.
- Write and evaluate triple integrals in cylindrical and spherical coordinates.

### Essential Questions

- What is a double integral, and how can it be used to find the surface area and volume of a solid?
- How does one write a double integral as an iterated integral?
- How does one use polar coordinates to simplify the integration of solids over circular regions?
- How can double integrals be used to calculate mass, electrical charge, center of mass, moments of inertia, and other physical attributes?
- What is a triple integral?

### Content (Scope and Sequence)

- Evaluating an iterated integral
- Using an iterated integral to find the area of a plane region
- Using a double integral to represent the volume of a solid region
- Evaluating a double integral as an iterated integral
- Finding the average value of a function over a region
- Writing and evaluating double integrals in polar coordinates
- Finding the mass and center of mass of a planar lamina using double integrals
- Finding moments of inertia using double integrals
- Using a double integral to find the area of a surface
- Using a triple integral to find the volume of a solid region
- Finding the center of mass and moments of inertia of a solid region
- Writing and evaluating a triple integral in cylindrical and spherical coordinates
- Using a Jacobian to change variable in a double integral

### Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

### Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.

3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

### **Assured Experiences**

Problem Sets

Homework

Mid-Unit Quizzes

Tests

### **Time Allocation**

Approximately 7 weeks

## Unit 5: Vector Analysis

### Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Calculus Curriculum Guide.

- Know how to sketch a vector field, determine whether the field is conservative, find a potential function, find curl, and find divergence.
- Use the Fundamental Theorem of Line Integrals.
- Understand and apply the theorems of Green, Stokes, and Gauss.

### Essential Questions

- What are some important properties of divergence and curl?
- How are double integrals and line integrals related?
- How does the Divergence Theorem relate triple integrals over a solid region and a surface integral over a surface?

### Content (Scope and Sequence)

- Determining whether a vector field is conservative
- Finding the curl and divergence of a vector field
- Understanding and using the concept of a piecewise smooth curve
- Writing and evaluating a line integral of a vector field
- Understanding and using the Fundamental Theorem of Line Integrals
- Using Green's Theorem to evaluate a line integral
- Understanding the definition of a parametric surface
- Finding a normal vector and tangent vector plane to a parametric surface
- Finding the area of a parametric surface
- Evaluating a surface integral as a double integral and for a parametric surface
- Understanding and using the Divergence Theorem to calculate flux
- Understanding and using Stokes' Theorem

### Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

### Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.

3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

### **Assured Experiences**

Problem Sets

Homework

Mid-Unit Quizzes

Tests

### **Time Allocation**

Approximately 7 weeks

## **Culminating Activity**

### **Final Exam**

The final exam is worth 10% of the student's Trumbull High School course grade.

## TEACHER GUIDE

### Unit 1: Vectors and the Geometry of Space

Sections:

- 11.1 Vectors in the Plane
- 11.2 Space Coordinates and Vectors in Space
- 11.3 The Dot Product of Two Vectors
- 11.4 The Cross Product of Two Vectors in Space
- 11.5 Lines and Planes in Space
- 11.6 Surfaces in Space
- 11.7 Cylindrical and Spherical Coordinates

### Unit 2: Vector-Valued Functions

- 12.1 Vector-Valued Functions
- 12.2 Differentiation and Integration of Vector-Valued Functions
- 12.3 Velocity and Acceleration
- 12.4 Tangent Vectors and Normal Vectors
- 12.5 Arc Length and Curvature

### Unit 3: Functions of Several Variables

- 13.1 Introduction of Several Functions of Several Variables
- 13.2 Limits and Continuity
- 13.3 Partial Derivatives
- 13.4 Differentials
- 13.5 Chain Rules for Functions of Several Variables
- 13.6 Directional Derivatives and Gradients
- 13.7 Tangent Lines and Normal Planes
- 13.8 Extrema of Two Functions of Two Variables
- 13.9 Applications of Extrema of Functions of Two Variables
- 13.10 Lagrange Multipliers

### Unit 4: Multiple Integration

- 14.1 Integrated Integrals and Area in the Plane
- 14.2 Double Integrals and Volume
- 14.3 Change of Variables: Polar Coordinates
- 14.4 Center of Mass and Moments of Inertia
- 14.5 Surface Area
- 14.6 Triple Integrals and Applications
- 14.7 Triple Integrals in Cylindrical and Spherical Coordinates
- 14.8 Change of Variables: Jacobians

### Unit 5: Vector Analysis

- 15.1 Vector Fields
- 15.2 Line Integrals
- 15.3 Conservative Vector Fields and Independence

- 15.4 Green's Theorem
- 15.5 Parametric Surfaces
- 15.6 Surface Integrals
- 15.7 Divergence Theorem
- 15.8 Stokes' Theorem

## **COURSE CREDIT**

One THS credit in Mathematics  
.8 University of Connecticut credits possible  
One class period daily for a full year

## **PREREQUISITES**

Completion of Advanced Placement / Early College Experience Calculus BC with a B or better and teacher recommendation.

## **TEXT**

Larson, Ron, and Bruce H. Edwards. *Calculus: AP Edition*. 9<sup>th</sup> ed. Boston: Brooks/Cole, 2010. Print.

## **SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY**

TI-84 Plus graphing calculators

## **CURRENT REFERENCES**

Fairfield Public Schools Multivariable Calculus Curriculum

<http://cdn.fairfieldschools.org/curriculum/math-2014/Multivariable%20Calculus%2061.pdf>

## **ASSURED STUDENT PERFORMANCE RUBRICS**

- Trumbull High School School-Wide Writing Rubric
- Trumbull High School School-Wide Problem-Solving Rubric
- Trumbull High School School-Wide Independent Learning and Thinking Rubric

## SCHOOL-WIDE RUBRICS

### Rubric 2: Write Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	<ul style="list-style-type: none"> <li>Establishes and maintains a clear purpose</li> <li>Demonstrates an insightful understanding of audience and task</li> </ul>	<ul style="list-style-type: none"> <li>Establishes and maintains a purpose</li> <li>Demonstrates an accurate awareness of audience and task</li> </ul>	<ul style="list-style-type: none"> <li>Establishes a purpose</li> <li>Demonstrates an awareness of audience and task</li> </ul>	<ul style="list-style-type: none"> <li>Does not establish a clear purpose</li> <li>Demonstrates limited/no awareness of audience and task</li> </ul>
Organization X_____	<ul style="list-style-type: none"> <li>Reflects sophisticated organization throughout</li> <li>Demonstrates logical progression of ideas</li> <li>Maintains a clear focus</li> <li>Utilizes effective transitions</li> </ul>	<ul style="list-style-type: none"> <li>Reflects organization throughout</li> <li>Demonstrates logical progression of ideas</li> <li>Maintains a focus</li> <li>Utilizes transitions</li> </ul>	<ul style="list-style-type: none"> <li>Reflects some organization throughout</li> <li>Demonstrates logical progression of ideas at times</li> <li>Maintains a vague focus</li> <li>May utilize some ineffective transitions</li> </ul>	<ul style="list-style-type: none"> <li>Reflects little/no organization</li> <li>Lacks logical progression of ideas</li> <li>Maintains little/no focus</li> <li>Utilizes ineffective or no transitions</li> </ul>
Content X_____	<ul style="list-style-type: none"> <li>Is accurate, explicit, and vivid</li> <li>Exhibits ideas that are highly developed and enhanced by specific details and examples</li> </ul>	<ul style="list-style-type: none"> <li>Is accurate and relevant</li> <li>Exhibits ideas that are developed and supported by details and examples</li> </ul>	<ul style="list-style-type: none"> <li>May contain some inaccuracies</li> <li>Exhibits ideas that are partially supported by details and examples</li> </ul>	<ul style="list-style-type: none"> <li>Is inaccurate and unclear</li> <li>Exhibits limited/no ideas supported by specific details and examples</li> </ul>
Use of Language X_____	<ul style="list-style-type: none"> <li>Demonstrates excellent use of language</li> <li>Demonstrates a highly effective use of standard writing that enhances communication</li> <li>Contains few or no errors. Errors do not detract from meaning</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrates competent use of language</li> <li>Demonstrates effective use of standard writing conventions</li> <li>Contains few errors. Most errors do not detract from meaning</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrates use of language</li> <li>Demonstrates use of standard writing conventions</li> <li>Contains errors that detract from meaning</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrates limited competency in use of language</li> <li>Demonstrates limited use of standard writing conventions</li> <li>Contains errors that make it difficult to determine meaning</li> </ul>

### Rubric 3: Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task	Student demonstrates sufficient understanding of the problem and most of the complexities of the task	Student demonstrates some understanding of the problem but requires assistance to complete the task	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal	Student gathers sufficient information from multiple sources including digital, print, and interpersonal	Student gathers some information from few sources including digital, print, and interpersonal	Student gathers limited or no information
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

### Rubric 5: Independent Learners And Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	Presentation shows compelling evidence of an independent learner and thinker. Solution shows deep understanding of the problem and its components. Solution shows extensive and appropriate application of 21 <sup>st</sup> Century Skills.	Presentation shows clear evidence of an independent learner and thinker. Solution shows adequate understanding of the problem and its components. Solution shows adequate application of 21 <sup>st</sup> Century Skills.	Presentation shows some evidence of an independent learner and thinker. Solution shows some understanding of the problem and its components. Solution shows some application of 21 <sup>st</sup> Century Skills.	Presentation shows limited or no evidence of an independent learner and thinker. Solution shows limited or no understanding of the problem. Solution shows limited or no application of 21 <sup>st</sup> Century Skills.