

TRUMBULL PUBLIC SCHOOLS
Trumbull, Connecticut

ACP Algebra II

Mathematics Department

2015

Last revision date: 2004

Curriculum Writing Team

Fran Basbagill	Department Chairperson
Elizabeth Capobianco	Teacher
Nancy Ciborowski	Teacher
Kristin Ruggiero	Teacher

Jonathan S. Budd, Ph.D., Director of Curriculum, Instruction & Assessments

ACP Algebra II

Table of Contents

Core Values and Beliefs	3
Introduction.....	3
Philosophy	3
Course Description	5
Assessments.....	5
Goals	6
Unit P: Preparation for Algebra II.....	8
Unit 1: Functions and Inverse Functions.....	12
Unit 2: Polynomial Functions.....	15
Unit 3: Rational Expressions and Functions.....	18
Unit 4: Trigonometric Functions	21
Unit 5: Exponential and Logarithmic Functions	23
Unit 6: Inferential Statistics.....	26
Course Credit	28
Prerequisites	28
Text	28
Supplementary Materials/Resources/Technology	28
Current References	28
Syllabus	29
Rubrics.....	30

The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in its employment.

CORE VALUES AND BELIEFS

Trumbull High School community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Student will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION

Advanced College Preparatory Algebra II is designed for the student who has successfully completed ACP Algebra I and ACP Geometry. Students will build on their work with linear, quadratic, and exponential functions to extend their repertoire of functions to include polynomial, rational, and radical functions. Students will work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations including solving quadratic equations over the set of complex numbers and solve exponential equations using the properties of logarithms.

The ACP Algebra II Curriculum Guide was last updated in 2004. Since 2004, high school mathematics curricula have shifted based on the Connecticut Core Standards, so topics previously taught in one course have been moved to a different course. In this course, new topics have been added, such as transformations of functions, end behavior of functions, and a brief unit on statistics. Units on trigonometry and exponential and logarithmic functions have been expanded to include more complicated problems, graphing, and applications. Finally, a graphing calculator is now required and used throughout the course.

PHILOSOPHY

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.

The successful mathematician will:

- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

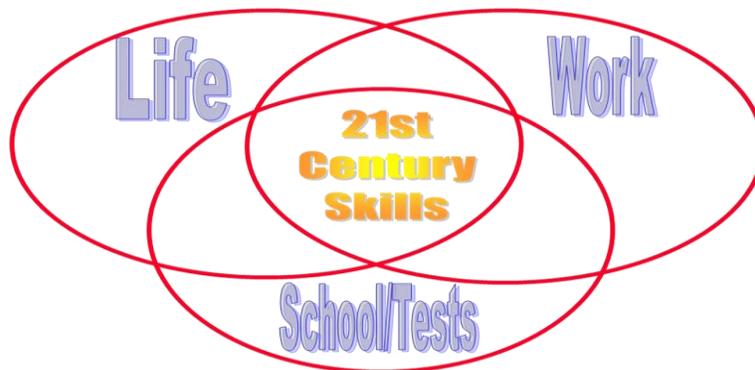
The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

General:

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourage students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media and Technology Skills

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals.
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE DESCRIPTION

Advanced College Preparatory Algebra II is designed to develop the eight standards of mathematical practices in students. Algebra II includes the study of functions and inverse functions; the analysis and building of functions; polynomials; complex numbers and analyzing functions using different representations; rational expressions and functions, with a focus on explaining reasoning; trigonometric functions; exponential and logarithmic functions, with an emphasis on constructing and comparing linear, and quadratic and exponential models. This course concludes with a brief look at statistics. A graphing calculator is required for this course.

ASSESSMENTS

Formative assessments will help guide student learning and summative assessments will test the students' level of mastery of knowledge and skills. Formal assessments will be various including, but not limited to, quizzes, problem sets, tests, and projects. The midterm and final exams are departmental exams and are given by each instructor who teaches the course.

GOALS

The Standards for Mathematical Practice describe varieties of expertise that all teachers of mathematics will develop in their students.

These practices rest on important “processes and proficiencies” that have long been valued in mathematics education.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit P: Preparation for Algebra II

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

8-EE Expressions and Equations

Analyze and solve linear equations and pairs of simultaneous linear equations

7. Solve linear equations in one variable.

- a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8-F Functions

Define, evaluate and compare functions

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

N-Q Quantities

Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- a. Factor a quadratic expression to reveal the zeroes of the function it defines.

A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .*

A-REI Reasoning with Equations and Inequalities

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)
11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear functions.

F-IF Interpreting Functions

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, [and] evaluate functions for inputs in their domains.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; [and] intervals where the function is increasing, decreasing, positive, or negative.*

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear functions and show intercepts.
 - e. Graph exponential functions, showing intercepts and end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{(t/10)}$, and classify them as representing exponential [functions].*

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions [and with exponential functions].
 - a. Prove that linear functions grow by equal differences over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

S-ID Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
 - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
 - c. Fit a linear function for a scatter plot that suggests a linear association.
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Essential Questions

- How can we use linear equations and linear inequalities to solve real-world problems?
- What is a solution set for a linear equation or linear inequality?
- What is a function?
- What is a linear function?
- What are the different ways that linear functions may be represented?
- What is the significance of a linear function's slope and y-intercept?
- How may linear functions model real-world situations?
- What does the number of solutions (none, one or infinite) of a system of linear equations represent?
- What characterizes exponential growth and decay?

Content (Scope and Sequence)

Determine if an equation is linear or exponential

Solve equations and inequalities

Evaluate expressions and functions

Write algebraic expressions to represent verbal expressions and vice versa

Find the midpoint of the segment connecting two points

Find distance between two points

Write the equation of a line

Find the x and y intercepts of a line

Graph a linear equation

Solve systems of equations

Factor expressions

Calculate the mean, median, mode, and IQR of a given set of data

Determine if there are outliers in a given set of data

Perform operations on polynomials

Simplify expressions using the laws of exponents

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Summer Packet

Department Exam

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 2 weeks

Unit One: Functions and Inverse Functions

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeroes when suitable factorizations are available, and showing end behavior.

A-CED Creating Equations

Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF Building Functions

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF Building Functions

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
 - c. Compose functions.
4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
 - b. Verify by composition that one function is the inverse of another.
 - c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
 - d. Produce an invertible function from a non-invertible function by restricting the domain.

A-REI Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Essential Questions

- How can we find the inverse of a function?
- How can we determine if two functions are inverse functions?
- When is a piece-wise function useful?
- How can a graphing calculator be used to evaluate differences between functions?

Content (Scope and Sequence)

Graph piece-wise functions

Graph the square root, cube root, absolute value and step functions

Find the inverse of a function

Test to see if two functions are inverse functions

Find intercepts

Find intervals of increase/decrease

Find relative maxima and minima (rely on graphing calculator for non-quadratic functions)

Discuss symmetry of a function

Discuss end behavior of a function

State domain and range

Function Composition

Transform Functions

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 4 weeks

Unit Two: Polynomial Functions

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

N-CN The Complex Number System

Perform arithmetic operations with complex numbers

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

A-REI Reasoning with Equations and Inequalities

4. Solve quadratic equations in one variable.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - c. Graph polynomial functions, identifying zeroes when suitable factorizations are available, and showing end behavior.

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Essential Questions

- How do we apply quadratic equations to real- life situations?
- How are factors, zeroes and x - intercepts of a quadratic function related?
- What are the different ways to solve a quadratic equation?
- How do you determine the most efficient method for solving a quadratic equation?

Content (Scope and Sequence)

Perform operations on complex numbers

Solve quadratic equations

Determine the nature of the roots using the discriminant

Graph polynomial functions

Find the zeroes of polynomials functions

Find the intercepts of polynomial functions

Find intervals of increase/decrease of polynomial functions

Find relative maxima and minima polynomial functions (using the calculator for non-quadratic functions)

Discuss the end behavior of polynomial functions

Complete the square

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 6 weeks

Unit Three: Rational Expressions and Functions

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

A-REI Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

Represent and solve equations and inequalities graphically

11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$*

A-APR Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
3. Represent constraints by equations or inequalities, and by systems of equations and /or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

F- IF Interpreting Functions

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-BF Building Functions

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Essential Questions

- What are extraneous solutions and how do they arise?
- What is the effect of varying the parameters a , h and k on the shape and position of $f(x) = a(x - h)^2 + k$?
- What is the effect of varying the parameters a , h and k on the shape and position of $f(x) = ab^{(x+h)} + k$?
- What is an asymptote and when they do occur?

Content (Scope and Sequence)

Solve radical equations

Solve rational equations

Even and odd functions

Graph $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and discuss the key features

Find the intersection of two functions using technology

Graph the solution set for a system of linear inequalities

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 4 weeks

Unit Four: Trigonometric Functions

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Analyze functions using different representations

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Interpret Functions that arise in applications in terms of the context

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-BF Building Functions

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

A-CED Creating Equations

Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Essential Questions

- How do trigonometric functions help to model periodic phenomena?
- How are special right triangles used to find the values of trigonometric functions on the unit circle?
- What is the difference between measuring an angle in degrees and radians?

Content (Scope and Sequence)

Graph the sine, cosine and tangent functions

Find the intercepts of the trigonometric graphs

Find the intervals of increase and decrease for the trigonometric graphs

Find the relative maxima and minima of the trigonometric graphs

Find the symmetry of the trigonometric graphs

Find the periodicity of the trigonometric graphs

Find the midline of the trigonometric graphs

Find the amplitude of the trigonometric graphs

Translate between degrees and radians

Use right triangle to calculate the trigonometric functions of angles on the unit circle

Introduce $\sin^2 \theta + \cos^2 \theta = 1$

Apply trigonometry to real- life situations

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 5 weeks

Unit Five: Exponential and Logarithmic Functions

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - b. Use the properties of exponents to interpret expressions for exponential functions.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF Building Functions

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
 - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

A-REI Reasoning with Equations and Inequalities

Represent and solve equations and inequalities graphically

11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

4. For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Essential Question

- How do exponential and logarithmic model real-world problems?

Content (Scope and Sequence)

Evaluate logarithms

Express exponential equations as logarithms

Express logarithms as exponential equations

Graph exponential growth and decay

Find the intercepts of exponential and logarithmic graphs

Find the intervals of increase and decrease of exponential and logarithmic graphs

Discuss the end behavior of exponential and logarithmic graphs

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 5 weeks

Unit Six: Inferential Statistics

Performance Standards

The Performance Standards align with the Connecticut Core Standards for Mathematics.

S-ID Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

S-IC Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Essential Questions

- How are measures of central tendency different from standard deviation?
- What are key characteristics of a normal distribution?
- Why is random sampling of a population done?
- How can an unbiased sample or an unbiased survey be conducted?
- How does the mean of sample compare to the mean of a population?

Content (Scope and Sequence)

Calculate standard deviation (with the graphing calculator)

Estimate population percentages for normal distributions

Recognize when it is appropriate to use a normal distribution

Develop margin of error

Compare treatment groups

Use simulations to make decisions

Instructional/Teaching Strategies

Brainstorming, Comprehension Questions based on instruction, Cuing Expected Behavior, Direct Instruction, Discussion Groups, Encouraging Students to Clarify & Expand Ideas, Peer Modeling, Question & Answer Sessions, Refocusing Students, Research using Technology, Restating & Rephrasing Key Concepts, Wait Time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Assured Questions

Evaluation/Assessment Methods

Tests, Quizzes, Problem Sets, Homework

Time Allocation

Approximately 6 weeks

COURSE CREDIT

One credit in Mathematics
One class period daily for a full year
Level: Advanced College Preparatory

PREREQUISITES

Grade of C or better in Algebra course 2014, grade of C or better in Geometry course 2024, and teacher recommendation.

TEXT

Algebra 2, Glencoe/McGraw Hill, 2003

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

A graphing calculator will be used extensively throughout class and is required for ACP Algebra.

CURRENT REFERENCES

Common Core State Standards - Mathematics

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Trumbull High School **Core Values and Beliefs Statement**

The Trumbull High School Community, which engages in an environment conducive to learning, believes that all students will READ and WRITE EFFECTIVELY, therefore communicating in an articulate and coherent manner. All students will participate in activities that address PROBLEM-SOLVING THROUGH CRITICAL THINKING. Students will use TECHNOLOGY as a tool in DECISION MAKING. We believe that by fostering self-confidence, self-directed and student-centered activities, we will encourage INDEPENDENT THINKING AND LEARNING. We believe ETHICAL CONDUCT to be paramount in sustaining our welcoming school climate.

Syllabus/Course Description

Course Name: Algebra II

Course Level: Advanced College Prep

Catalog #: 2034

Prerequisites:

Grade of C or better in Algebra course 2014, grade of C or better in Geometry course 2024 and teacher recommendation.

General Description of Course Content:

Algebra II is designed to develop the eight standards of mathematical practices in students. Algebra II includes the study of functions and inverse functions, the analysis and building of functions, polynomials, complex numbers and analyzing functions using different representations, rational expressions and functions, with a focus on explaining reasoning, trigonometric functions, exponential and logarithmic functions, with an emphasis on constructing and comparing linear, quadratic and exponential models. This course concludes with a brief look at statistics. A graphing calculator is required for this course.

Assessment:

Students are evaluated by their performance on teacher and textbook produced tests and quizzes, departmental midyear and final exams.

Text and Supplementary Materials:

- 1). Algebra 2; Glencoe McGraw Hill
- 2). Teacher-prepared worksheets
- 3). Graphing calculators

RUBRICS

Rubric 2: Writes Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	<ul style="list-style-type: none"> Establishes and maintains a clear purpose. Demonstrates an insightful understanding of audience and task. 	<ul style="list-style-type: none"> Establishes and maintains a purpose. Demonstrates an accurate awareness of audience and task. 	<ul style="list-style-type: none"> Establishes a purpose. Demonstrates an awareness of audience and task. 	<ul style="list-style-type: none"> Does not establish a clear purpose. Demonstrates limited/no awareness of audience and task.
Organization X_____	<ul style="list-style-type: none"> Reflects sophisticated organization throughout. Demonstrates logical progression of ideas. Maintains a clear focus. Utilizes effective transitions. 	<ul style="list-style-type: none"> Reflects organization throughout. Demonstrates logical progression of ideas. Maintains a focus. Utilizes transitions. 	<ul style="list-style-type: none"> Reflects some organization throughout. Demonstrates logical progression of ideas at times. Maintains a vague focus. May utilize some ineffective transitions. 	<ul style="list-style-type: none"> Reflects little/no organization. Lacks logical progression of ideas. Maintains little/no focus. Utilizes ineffective or no transitions.
Content X_____	<ul style="list-style-type: none"> Is accurate, explicit, and vivid. Exhibits ideas that are highly developed and enhanced by specific details and examples. 	<ul style="list-style-type: none"> Is accurate and relevant. Exhibits ideas that are developed and supported by details and examples. 	<ul style="list-style-type: none"> May contain some inaccuracies. Exhibits ideas that are partially supported by details and examples. 	<ul style="list-style-type: none"> Is inaccurate and unclear. Exhibits limited/no ideas supported by specific details and examples.
Use of Language X_____	<ul style="list-style-type: none"> Demonstrates excellent use of language. Demonstrates a highly effective use of standard writing that enhances communication. Contains few or no errors. Errors do not detract from meaning. 	<ul style="list-style-type: none"> Demonstrates competent use of language. Demonstrates effective use of standard writing conventions. Contains few errors. Most errors do not detract from meaning. 	<ul style="list-style-type: none"> Demonstrates use of language. Demonstrates use of standard writing conventions. Contains errors that detract from meaning. 	<ul style="list-style-type: none"> Demonstrates limited competency in use of language. Demonstrates limited use of standard writing conventions. Contains errors that make it difficult to determine meaning.

Rubric 3 - Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task.	Student demonstrates sufficient understanding of the problem and most of the complexities of the task.	Student demonstrates some understanding of the problem but requires assistance to complete the task.	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task.
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal.	Student gathers sufficient information from multiple sources including digital, print, and interpersonal.	Student gathers some information from few sources including digital, print, and interpersonal.	Student gathers limited or no information.
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies.	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies.	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan.
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

Rubric 5: Independent Learners and Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	<p>Presentation shows compelling evidence of an independent learner and thinker.</p> <p>Solution shows deep understanding of the problem and its components.</p> <p>Solution shows extensive and appropriate application of 21st Century Skills.</p>	<p>Presentation shows clear evidence of an independent learner and thinker.</p> <p>Solution shows adequate understanding of the problem and its components.</p> <p>Solution shows adequate application of 21st Century Skills.</p>	<p>Presentation shows some evidence of an independent learner and thinker.</p> <p>Solution shows some understanding of the problem and its components.</p> <p>Solution shows some application of 21st Century Skills.</p>	<p>Presentation shows limited or no evidence of an independent learner and thinker.</p> <p>Solution shows limited or no understanding of the problem.</p> <p>Solution shows limited or no application of 21st Century Skills.</p>