

TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

PreCalculus

**Advanced College-Preparatory
Mathematics Department**

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The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in any of its programs.

CORE VALUES AND BELIEFS

The Trumbull High School community engages in an environment conducive to learning which believes that all students will **read and write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION & PHILOSOPHY

ACP PreCalculus is designed for students who have successfully completed ACP or Honors Algebra II. The content and skills in PreCalculus enable students to prepare for a Calculus course, either in high school or in college. After successful completion of ACP PreCalculus, students will be prepared to enroll in ACP Calculus or AP Calculus AB, as well as other senior mathematics electives.

The study of functions and their characteristics serves as the theme for this course. In particular, polynomial, rational, exponential, logarithmic, and trigonometric functions, and their applications, are examined. Students will learn to solve equations and develop models to help explore practical applications. Students will also preview calculus in finding limits using a table, graph, and algebraic techniques and in finding the derivative of a function.

The textbook, *Precalculus: Enhanced with Graphing Utilities* by Sullivan and Sullivan (2017), offers a plethora of features to assist students in learning vocabulary, problem-solving, skill-building, and engaging in applications and extensions. The pacing and sequence of the curriculum is suited to meet the content standards and the eight mathematical practices of the 2010 Connecticut Core Standards for Mathematics.

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.

The successful mathematician will:

- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively

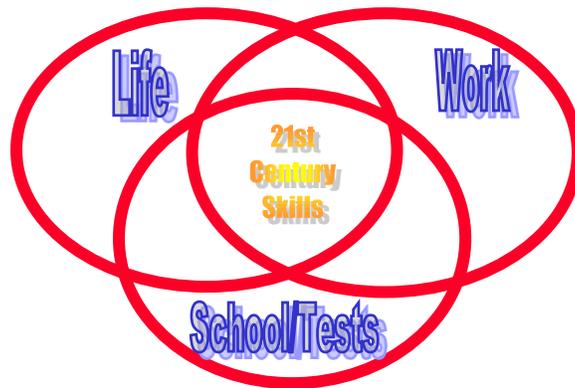
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourage students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media and Technology Skills

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE GOALS

The following Course Goals derive from the 2010 Connecticut Core Standards for Mathematical Practice, which describe varieties of expertise that all teachers of mathematics will develop in their students. These practices rest on important “processes and proficiencies” that have long been valued in mathematics education.

At the completion of this course, students will:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They

routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The following Course Goals derive from the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

5b. Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

- 5c. Students break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

COURSE ENDURING UNDERSTANDINGS

Students will understand that . . .

- various aspects of polynomial functions, rational functions, exponential functions, and logarithmic functions occur frequently in a variety of applications.
- the six trigonometric functions have fundamental relationships in their value in portraying natural phenomena.
- limits, continuity, and the derivative of a function have increased relevance in advanced mathematics.

COURSE ESSENTIAL QUESTIONS

- What are the properties of polynomial, rational, exponential, and logarithmic functions, and how can these functions be used to solve applied problems?
- How can the development of the trigonometric functions be explored through two approaches: the use of right triangles and the unit circle?
- How can trigonometric functions and their inverses be used to solve problems?
- What is a limit, and how can it be interpreted?

COURSE KNOWLEDGE & SKILLS

Students will know . . .

- polynomial, rational, exponential, and logarithmic functions and their graphs.
- the six trigonometric functions and the unit circle.
- the formal definitions, applications, and properties of limits.

Students will be able to . . .

- analyze and graph a rational function.
- use properties of exponents and logarithms to solve equations.
- define the six trigonometric functions, explore their domain and range, and develop a list of their properties.
- use trigonometric identities to simplify trigonometric expressions.
- use trigonometric functions to solve applied problems involving right and oblique triangles.
- find the limit of a function from a table, from a graph, and from using algebraic techniques.
- find the derivative of a function.

COURSE SYLLABUS

Course Name

ACP PreCalculus

Level

Advanced College-Preparatory

Prerequisites

Grade of C+ or better in ACP Algebra II, or teacher recommendation

Materials Required

TI-84 Plus graphing calculator

General Description of the Course

The study of functions and their characteristics serves as the theme for this course. In particular, polynomial, rational, exponential, logarithmic, and trigonometric functions, and their applications, are examined. Students will learn to solve equations and develop models to help explore practical applications. Students will also preview calculus in finding limits using a table, graph, and algebraic techniques and in finding the derivative of a function. A graphing calculator is required for this course. This course is intended for students who will take ACP Calculus or AP Calculus AB.

Assured Assessments

Formative Assessments:

Formative assessments can include, but are not limited to:

- Warm-up problems (Units 1, 2, 3, 4, 5, 6, 7, 8)
- Exit slips (Units 1, 2, 3, 4, 5, 6, 7, 8)
- Group problem sets (Units 1, 2, 3, 4, 5, 6, 7, 8)
- Homework (Units 1, 2, 3, 4, 5, 6, 7, 8)
- Checkpoint quizzes (Units 1, 2, 3, 4, 5, 6, 7, 8)

Summative Assessments:

- Common assessments (Units 1, 2, 3, 4, 5, 6, 7, 8)
- Small-group collaborative poster project (Unit 3)
- Common project based on real-world data and phenomena (Unit 6)
- Common midterm examination
- Common final examination

Core Text

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

UNIT 1

Graphs

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

The following Unit Goals are TPS-created, and influenced by the Fairfield Public Schools PreCalculus Curriculum Guide.

- Represent and analyze functional relationships using a variety of strategies, tools, and technologies.
- Express quantitative relationships numerically in multiple ways in order to make connections and simplify calculations.
- Test an equation for symmetry, find the intercepts of an equation algebraically, and solve equations algebraically and/or using a graphing utility.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers and modeled graphically?
- How are equations solved and represented in a multitude of ways?

Scope and Sequence

1. Distance and midpoint formulas
2. Intercepts of an equation
3. Tests for symmetry
4. Solving equations algebraically and using a graphing calculator
5. Equations of lines
6. Equations and graphs of circles

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 14 school days

UNIT 2

Functions and Their Graphs

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF Building Functions

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What are the properties of a function, and how are functions used in applications?

- What is the effect on the graph of a function when specific values of the function are changed?
- How is the average rate of change of a function used in finding the equation of a secant line?

Scope and Sequence

1. Properties of functions
2. Difference quotient of a function
3. Average rate of change of a function
4. Equation of a secant line
5. Even and odd functions
6. Library of functions
7. Piecewise-defined functions and the related domain and range
8. Transformations of functions

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 24 school days

UNIT 3

Polynomial and Rational Functions

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

A-APR Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How are polynomial functions and rational functions related?
- What are the properties of polynomial functions and rational functions?
- How does the graph of a rational function relate to its properties?

Scope and Sequence

1. Polynomial functions and models
2. Properties of rational functions
3. Graph of a rational function
4. Polynomial and rational inequalities

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. In addition, students will complete a small-group collaborative poster project on graphing a rational function; students will be assessed via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 21 school days

UNIT 4

Exponential and Logarithmic Functions

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-BF Building Functions

Build new functions from existing functions.

4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.
 - b. (+) Verify by composition that one function is the inverse of another.
 - c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- When and how can composition be used in constructing and analyzing functions?
- How are the properties of exponential functions used to solve exponential equations?
- How are exponential and logarithmic functions related?

Scope and Sequence

1. Composite functions
2. Inverse functions
3. Exponential functions
4. Logarithmic functions
5. Properties of logarithms
6. Exponential and logarithmic equations

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be

administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 15 school days

UNIT 5

Applications of Trigonometric Functions

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

G-SRT Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What the six trigonometric ratios?
- How are the six trigonometric ratios related to each other?
- How are the properties of right triangles and trigonometry used in the real world?
- How is the Complementary Angle Theorem used to find the exact value of a trigonometric expression?
- How are inverse trigonometric ratios used to find angle measures of a right triangle?
- How are the Laws of Sines and Cosines used in finding side and angle measures in an oblique triangle?
- Given SSA, what determines if there can be more than one solution in the triangle?
- How is trigonometry used to find the area of a triangle?

Scope and Sequence

1. Right-triangle trigonometry
2. Complementary Angle Theorem
3. Inverse trigonometric functions

4. Applications of trigonometric functions
5. Law of Sines
6. Law of Cosines
7. Area of oblique triangles

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 7 school days

UNIT 6

Trigonometric Functions

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- How are the properties of special right triangles relevant to the unit circle?
- How can the trigonometric functions be represented with right triangles in circles?
- What are the fundamental relationships between and among the trigonometric functions?
- How do the properties of transformation apply to the trigonometric functions?
- How do the features of a trigonometric function correspond to various natural phenomena?

Scope and Sequence

1. Converting from degrees to radians and radians to degrees
2. Finding exact values of the trigonometric functions using a point on the unit circle
3. Finding exact values of the trigonometric functions of $\pi/3$, $\pi/4$, and $\pi/6$

4. Determining the domain and range of the trigonometric functions
5. Determining the period of the trigonometric functions
6. Finding the values of the trigonometric functions using fundamental identities
7. Using even-odd properties to find exact values of the trigonometric functions
8. Graphing sine and cosine functions using transformations
9. Graphing sine and cosine functions using key points
10. Determining the amplitude and period of sinusoidal functions
11. Finding an equation for a sinusoidal graph
12. Identifying graphs of tangent, cotangent, cosecant, and secant functions
13. Building sinusoidal models from real-world data and phenomena

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common assessment scored via a common scoring guide, and will also complete a common project based on real-world data and phenomena and scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 28 school days

UNIT 7

Analytic Trigonometry

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

F-TF Trigonometric Functions

Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What are the inverse functions?
- What types of relationships do inverse functions have with other functions?
- How are the domain and range of inverse functions related?
- How are exact values of trigonometric functions found?

Scope and Sequence

1. Inverse trigonometric functions
2. Solving equations involving a single trigonometric function
3. Solving trigonometric equations using fundamental identities
4. Using algebra to simplify trigonometric expressions
5. Establishing identities
6. Using sum and difference formulas to find exact values
7. Using sum and difference formulas to establish identities
8. Using sum and difference formulas involving inverse trigonometric functions
9. Solving trigonometric equations linear in sine and cosine
10. Using double-angle formulas to find exact values

11. Using double-angle formulas to establish identities
12. Using half-angle formulas to find exact values

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 34 school days

UNIT 8

A Preview of Calculus: The Limit

Unit Goals

At the completion of this unit, students will:

The following Unit Goals are TPS-created, and influenced by the Fairfield Public Schools PreCalculus Curriculum Guide.

- Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity.
- Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
- Use graphing calculators to verify and estimate limits.
- Prove and use special limits, such as the limits of $\left(\frac{\sin(x)}{x}\right)$ and $\left(\frac{1-\cos(x)}{x}\right)$, as x tends to 0.

The following Unit Goal aligns with the 2016 International Society for Technology in Education Standards.

ISTE Computational Thinker
(Standard 5)

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

Unit Essential Questions

- What is a limit?
- How does continuity depend on limits?
- What are the different ways that a limit can fail to exist?
- How does one evaluate a limit using properties of limits?
- How is continuity determined at a point and on an open interval?
- How does one determine one-sided limits and continuity on a closed interval?
- How does one determine the derivative of a function?

Scope and Sequence

1. Finding limits using tables and graphs
2. Finding limits analytically and algebraically
3. Continuity and one-sided limits
4. Finding an equation of the tangent line to the graph of a function
5. Finding the derivative of a function

Assured Assessments

Formative Assessment:

Students will participate in various warm-up problems, exit slips, group problem sets, homework, and self-reflections throughout the unit. Checkpoint quizzes will also be administered to assess students' understanding of essential concepts and skills. See "Teacher Guide," p. 26.

Summative Assessment:

Students will take a common end-of-unit assessment scored via a common scoring guide. See "Teacher Guide," p. 26.

Resources

Core

- Sullivan, Michael, and Michael Sullivan III. *Precalculus: Enhanced with Graphing Utilities*. 7th ed. New York: Pearson, 2017. Print.

Supplemental

- Online instructor resources including Instructor's Solution Manual, PowerPoint Presentations, TestGen Software, and MyMathLab

Time Allotment

- Approximately 15 school days

TEACHER GUIDE

Unit One: Graphs

Sections:

- 1.1 The Distance and Midpoint Formulas
- 1.2 Intercepts; Symmetry; Graphing Key Equations
- 1.3 Solving Equations Using a Graphing Utility
- 1.4 Lines
- 1.5 Circles

Unit Two: Functions and Their Graphs

- 2.1 Functions
- 2.2 The Graph of a Function
- 2.3 Properties of Functions
- 2.4 Library of Functions; Piecewise-Defined Functions
- 2.5 Graphing Techniques: Transformations

Unit Three: Polynomial and Rational Functions

- 4.1 Polynomial Functions and Models
- 4.4 Properties of Rational Functions
- 4.5 The Graph of a Rational Function
- 4.6 Polynomial and Rational Inequalities

Unit Four: Exponential and Logarithmic Functions

- 5.1 Composite Functions
- 5.2 One-to-One Functions; Inverse Functions
- 5.3 Exponential Functions
- 5.4 Logarithmic Functions
- 5.5 Properties of Logarithms
- 5.6 Logarithmic and Exponential Equations

Unit Five: Applications of Trigonometric Functions

- 8.1 Right-Triangle Trigonometry; Applications
- 8.2 Law of Sines
- 8.3 Law of Cosines
- 8.4 Area of a Triangle

***Midterm exam**

Unit Six: Trigonometric Functions

- 6.1 Angles and Their Measure
- 6.2 Trigonometric Functions: The Unit Circle Approach
- 6.3 Properties of the Trigonometric Functions
- 6.4 Graphs of the Sine and Cosine Functions
- 6.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions
- 6.6 Phase Shift; Sinusoidal Curve Fitting

Unit Seven: Analytic Trigonometry

- 7.1 The Inverse Sine, Cosine, and Tangent Functions
- 7.2 The Inverse Trigonometric Functions (Continued)
- 7.3 Trigonometric Equations
- 7.4 Trigonometric Identities
- 7.5 Sum and Difference Formulas
- 7.6 Double-Angle and Half-Angle Formulas

Unit Eight: A Preview of Calculus: The Limit

- 14.1 Finding the Limits Using Tables and Graphs
- 14.2 Algebra Techniques for Finding Limits
- 14.3 One-Sided Limits; Continuous Functions
- 14.4 The Tangent Problem; The Derivative

***Final exam**

Timeline	Unit of Study	Assessments <i>CA = Common Assessment</i> <i>A = Teacher-Designed Assessment</i>
1st Marking Period		
Unit 1 (14 days)	Summer Packet Review & 1.1 & 1.4 Chapter 1: 1.2, 1.3, 1.5 (obj. 3)	CA: Summer Packet Review 1.1 & 1.4 A: 1.2, 1.3, & 1.5
Unit 2 (24 days)	Chapter 2: 2.1 (objs. 3-5), 2.2, 2.3, 2.4 (omitting greatest integer), 2.5	A: 2.1 & 2.2 A: 2.3 CA: 2.1 – 2.5
Unit 3 Part 1 (5 days)	Chapter 4: 4.1	A: 4.1
2nd Marking Period		
Unit 3 Part 2 (16 days)	Chapter 4: 4.4, 4.5 (obj. 1), 4.6	CA: 4.4 & 4.5 A: Project 4.4 & 4.5 A: 4.6
Unit 4 (15 days)	Chapter 5: 5.1, 5.2, 5.3, 5.4, 5.5, 5.6	A: 5.1 & 5.2 CA: 5.3 – 5.6
Unit 5 (7 days)	Chapter 8: 8.1, 8.2, 8.3, 8.4	CA: 8.1 – 8.4
3 days	Midterm Review	
3rd Marking Period		
Unit 6 (28 days)	Chapter 6: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6	CA: 6.1 & 6.2 A: Unit Circle A: 6.3 CA: 6.4 & 6.6 CA: Weather Project (6.4 & 6.6)
Unit 7 Part 1 (14 days)	Chapter 7: 7.1, 7.2, 7.3	A: 7.1 and 7.2 A: 7.3
4th Marking Period		
Unit 7 Part 2 (20 days)	Chapter 7: 7.4, 7.5, 7.6	A: 7.4 CA: 7.5 & 7.6
Unit 8 (15 days)	Chapter 14: 14.1, 14.2, 14.3 (include $\lim_{x \rightarrow \infty}$), 14.4 (objs. 1 & 2)	A: 14.1 A: 14.2 A: 14.3 – 14.4 CA: 14.1 – 14.4
4 days	Final Exam Review	

COURSE CREDIT

One credit in Mathematics
One class period daily for a full year

PREREQUISITES

Grade of C+ or better in ACP Algebra II, or teacher recommendation.

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department- and teacher-prepared materials

TI-84 Plus graphing calculators

CURRENT REFERENCES

2010 Connecticut Core Standards for Mathematics

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric (attached)
- Trumbull High School School-Wide Problem-Solving Rubric (attached)
- Trumbull High School School-Wide Independent Learning and Thinking Rubric (attached)

Trumbull High School School-Wide Writing Rubric

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	<ul style="list-style-type: none"> • Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task 	<ul style="list-style-type: none"> • Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task 	<ul style="list-style-type: none"> • Establishes a purpose • Demonstrates an awareness of audience and task 	<ul style="list-style-type: none"> • Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_____	<ul style="list-style-type: none"> • Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions 	<ul style="list-style-type: none"> • Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions 	<ul style="list-style-type: none"> • Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions 	<ul style="list-style-type: none"> • Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_____	<ul style="list-style-type: none"> • Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples 	<ul style="list-style-type: none"> • Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples 	<ul style="list-style-type: none"> • May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples 	<ul style="list-style-type: none"> • Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_____	<ul style="list-style-type: none"> • Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning 	<ul style="list-style-type: none"> • Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors. Most errors do not detract from meaning 	<ul style="list-style-type: none"> • Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning 	<ul style="list-style-type: none"> • Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Trumbull High School School-Wide Problem-Solving Rubric

Category/ Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	<ul style="list-style-type: none"> Student demonstrates clear understanding of the problem and the complexities of the task 	<ul style="list-style-type: none"> Student demonstrates sufficient understanding of the problem and most of the complexities of the task 	<ul style="list-style-type: none"> Student demonstrates some understanding of the problem but requires assistance to complete the task 	<ul style="list-style-type: none"> Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	<ul style="list-style-type: none"> Student gathers compelling information from multiple sources including digital, print, and interpersonal 	<ul style="list-style-type: none"> Student gathers sufficient information from multiple sources including digital, print, and interpersonal 	<ul style="list-style-type: none"> Student gathers some information from few sources including digital, print, and interpersonal 	<ul style="list-style-type: none"> Student gathers limited or no information
Reasoning and Strategies X_____	<ul style="list-style-type: none"> Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies 	<ul style="list-style-type: none"> Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies 	<ul style="list-style-type: none"> Student demonstrates some critical thinking skills to develop a plan integrating some strategies 	<ul style="list-style-type: none"> Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	<ul style="list-style-type: none"> Solution shows deep understanding of the problem and its components Solution shows extensive use of 21st-century technology skills 	<ul style="list-style-type: none"> Solution shows sufficient understanding of the problem and its components Solution shows sufficient use of 21st-century technology skills 	<ul style="list-style-type: none"> Solution shows some understanding of the problem and its components Solution shows some use of 21st-century technology skills 	<ul style="list-style-type: none"> Solution shows limited or no understanding of the problem and its components Solution shows limited or no use of 21st-century technology skills

Trumbull High School School-Wide Independent Learning and Thinking Rubric

Category/ Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	<ul style="list-style-type: none"> • Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work 	<ul style="list-style-type: none"> • Student demonstrates initiative by generating appropriate questions, creating original projects/work 	<ul style="list-style-type: none"> • Student demonstrates some initiative by generating questions, creating appropriate projects/work 	<ul style="list-style-type: none"> • Student demonstrates limited or no initiative by generating few questions and creating projects/work
Independent Research & Development X_____	<ul style="list-style-type: none"> • Student is analytical, insightful, and works independently to reach a solution 	<ul style="list-style-type: none"> • Student is analytical, and works productively to reach a solution 	<ul style="list-style-type: none"> • Student reaches a solution with direction 	<ul style="list-style-type: none"> • Student is unable to reach a solution without consistent assistance
Presentation of Final Product X_____	<ul style="list-style-type: none"> • Presentation shows compelling evidence of an independent learner and thinker • Solution shows deep understanding of the problem and its components • Solution shows extensive and appropriate application of 21st-century skills 	<ul style="list-style-type: none"> • Presentation shows clear evidence of an independent learner and thinker • Solution shows adequate understanding of the problem and its components • Solution shows adequate application of 21st-century skills 	<ul style="list-style-type: none"> • Presentation shows some evidence of an independent learner and thinker • Solution shows some understanding of the problem and its components • Solution shows some application of 21st-century skills 	<ul style="list-style-type: none"> • Presentation shows limited or no evidence of an independent learner and thinker • Solution shows limited or no understanding of the problem and its components • Solution shows limited or no application of 21st-century skills